

# Measuring EROI (energy return on investment) on a national level: two proposed approaches

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## Abstract

Concerns have been raised that declining EROI (energy return on energy investment) from fossil fuels and low levels of EROI for alternative energy sources could constrain the ability of economies to continue to deliver economic growth and improvements in social wellbeing. This paper contributes to the understanding of such concerns through the development of two methodologies to calculate a national-level EROI. The first is a physical approach using a novel application of EROI data generated through a system dynamics model. The second is a mixed physical and monetary approach using input-output analysis. In developing these new EROI approaches, the paper analyses the policy relevance of a national-level EROI. It also discusses the remaining conceptual and methodological issues relating to defining EROI for a national economy. Finally, it describes the methodology and data requirements for each approach and will present some initial results at the ESEE 2015 conference.

## 1. Introduction

The concept of energy return on energy investment (EROI) is part of the broad field of study of net energy analysis (NEA), and is one way of measuring and comparing the net energy gains from different energy sources and processes. Building on a long history of ideas in biophysical economics (Cleveland, 1987), this concept has been used by e.g. Hall and Klitgaard (2012) as a basis for further developing an energy focused approach to the economy. This energy focused approach is driven by concerns relating to depletion effects outweighing technological advancements for fossil fuel extraction –reducing their EROI- and currently comparatively low EROI values of renewable energy technologies that are being increasingly exploited to address climate change concerns (Hall, Lambert, & Balogh, 2014).

The higher the EROI of an energy supply technology, the more valuable it is in terms of producing (economically) useful output. An analysis of the EROI of different fuels and conversion processes using particular technologies has been made in e.g. Cleveland (2005), Brandt (2011), Hall, Lambert, & Balogh (2014). However, less attention has so far been paid to determining EROI values for national economies. A national-level EROI requires a different approach due to the mix of particular resource locations, exploitation time and technologies applied to provide the required energy supply. Cross-border issues also need to be addressed i.e. the energy imports from, and energy exports to, other countries.

This paper thus aims to contribute to clarifying some of the methodological debates relating to measuring EROI for national economies, and also to relating EROI to exergy and useful work analyses of national economies. It does so by proposing two initial methodological approaches. These can be broadly summarised as a system-dynamics physical approach and an Input-Output (IO) mixed (physical and monetary) approach. The national-level EROI perspective is important for investigating the relations between resource depletion and

technological progress in low carbon scenarios, and it can help inform policy decisions that aim to manage an energy transition (Carbajales-Dale, Barnhart, Brandt, & Benson, 2014).

## 2. Why is National-level EROI relevant?

Traditional energy analyses do not address directly the issue of resource depletion (or reduced accessibility) in terms of the increasing need of energy expenditure to support our energy needs for economic activities at a national level. It is in this respect that a national-level EROI becomes relevant as another tool in the energy analysis toolbox. As it is defined here, a national-level EROI studies extraction/capture process (the very initial part of the energy chain), hence it can be thought of as a necessary complement of energy analysis that focus on subsequent parts of the energy chain.

We argue that the analysis in this paper is complementary to approaches analysing exergy efficiency of national economies (Ayres & Warr, 2009; Brockway, Barrett, Foxon, & Steinberger, 2014). Exergy efficiency analysis measures losses at each stage of energy conversion processes relating to the ability to produce useful work outputs from primary energy (exergy) inputs. EROI on the other hand, is concerned with the ratio between the energy returned from an energy resource exploitation process and the energy that has to be invested in producing that energy.

When measured over time, EROI can provide information about the extent of resource depletion and technological change, whilst exergy and useful work analysis relates to conversion efficiency and energy quality. Therefore, if the system boundary for EROI is established at the extraction/capture level (rather than including downstream transformation processes), a national-level EROI time series can be analysed together with exergy and useful work time series to understand how the dynamics of resource depletion (or accessibility) and technological change relate to energy quality and the dynamics of conversion efficiencies. I.e. are upward trends in conversion efficiency counter balancing downward trends in resource depletion?

EROI has economic relevance since the energy return in excess of all the energy inputs facilitates other economic activities. EROI depends on temporal factors such as the extent of depletion or utilisation of a resource and spatial factors determining both the quantity and quality of the resource. The type and development of technology deployed will also influence the EROI at any particular point in time. EROI (over time) can be used to determine to what extent technological advancement is outweighing resource depletion and vice versa (Murphy, Hall, Dale, & Cleveland, 2011). This paper explores these issues for national economies, and thus also addresses questions of aggregation between different energy resources at a national level.

As a final thought, in 1974 the US passed a law such that “all prospective energy supply technologies considered for commercial application must be assessed and evaluated in terms of their ‘potential for production of net energy’” (Berndt, 1982), which was triggered by the 1973-74 oil crisis and abandoned after normality resumed. Perhaps it is time to reinstate similar laws, but for the guidance of national energy policy, where  $EROI_{nat}$  can

help inform policy decisions that aim to manage the energy transition (Carbajales-Dale et al., 2014).

### 3. EROI Concepts and Methodology

#### 3.1. EROI concepts

EROI (or EROEI) stands for energy return on (energy) invested and is a key metric in Net Energy Analysis (NEA). The concept was first introduced by Charles Hall in the 1970s in his PhD thesis (Hall, 1972) and in subsequent journal papers e.g. Hall, Lavine, & Sloane (1979). The term (EROI) however, was first used by Cleveland et al. (1984). It is a dimensionless ratio and it is usually defined as “the ratio of energy returned from an energy-gathering activity compared to the energy invested in that process” (Hall & Kiltgaard, 2012, p. 310).

Most EROI studies consider an energy supply technology for a particular resource type and in a particular location. Such studies typically have the “mine-mouth” (or “well-head” or “farm-gate”) as the boundary drawn for evaluating the energy returned to society in relation to the energy required to get it, without further processing (Murphy & Hall, 2010). The calculation of EROI is then the ratio of energy output from the extraction stage (numerator) e.g. oil, and the direct and indirect energy and material inputs (measured in embodied energy) required to obtain that energy output (denominator) e.g. energy consumed operating the oil rig and energy to make the oil rig and associated materials (Hall & Kiltgaard, 2012; Murphy et al., 2011).

$$EROI_{stnd} = \frac{\text{energy output from extraction}}{\text{direct and indirect energy inputs}} \quad (1)$$

This class of EROI calculations are referred to as “standard” EROI ( $EROI_{stnd}$ ) (Murphy et al., 2011). A graphical description can be found in Figure 1, but note that  $EROI_{stnd}$  here is referred to as  $EROI_{st}$ .

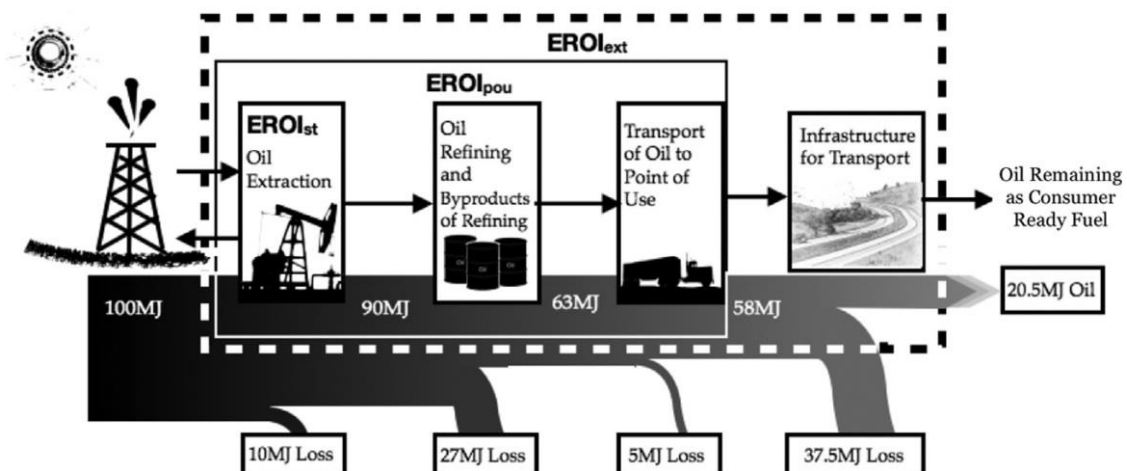


Figure 1. Types of EROI

Source: Taken from Hall et al. (2014, p. 142).

$EROI_{std}$  is particularly useful when comparing different fuels or energy carriers, or when analysing changes in EROI of a specific fuel over time (Lambert, Hall, Balogh, Gupta, & Arnold, 2014). For example, Murphy and Hall (2011) analyse how a continuously declining EROI for oil over the last 40 years can have seriously adverse consequences on the wider economy due to oil dependence. Hall et al. (2014) further warn of the large economic consequences of declining EROIs of all fossil fuels over the last two decades, considering that fossil fuels still represent over 80% of global primary energy supply (IEA, 2014).

EROI has also been associated to the energy quality of the different energy sources, in the sense that a higher EROI value means that a bigger proportion of the energy delivered to society can go into economic activities other than to the energy sector and vice versa (Murphy et al., 2011). In other words EROI is a measure of economic potential, rather than allowing for the complex physical, technical and social attributes of energy quality. Thus, the EROI of a certain resource over time can help to indicate the capability for economic expansion rather than being a comprehensive measure of energy quality.

But perhaps more importantly, EROI has a say in relation to resource depletion and technological change, both of which are very relevant for the economic system, particularly from an ecological economics perspective. A declining EROI in time means that resource depletion is outweighing technological change (Murphy et al., 2011), i.e. the amount of a certain energy resource (or its accessibility Dale, Krumdieck, & Bodger, 2012a) is declining faster than the advancements in technology to harvest it more efficiently. The consideration of these two factors in the calculation of EROI was applied in the development of a dynamic EROI function included in a simulation model of the interaction between the energy sector and the wider economy called “GEMBA” – Global Energy Model a Biophysical Approach (Dale, Krumdieck, & Bodger, 2011, 2012b).

Other, less common, types of EROI (besides  $EROI_{std}$ ) vary depending on the chosen system boundary. It should be noted that whichever boundary is applied it is important that consistency is maintained in the accounting for energy output and energy inputs i.e. that the numerator and the denominator relate to each other. For example, if the system boundary is expanded to include the conversion from primary energy to energy carrier, the processing stage is included in the boundary drawn for energy outputs (numerator) and the direct and indirect energy and materials needed to carry out the processing should be added to the energy inputs (denominator).

Similarly, if the system boundary is expanded to include the delivery of energy carriers to the point of use, the energy delivered to the final users is included in the boundary for energy outputs (numerator) and the direct and indirect energy and materials needed to carry out the delivery should be added to the energy inputs (denominator). This is referred to as  $EROI_{pou}$  in the illustration of the energy chain in Figure 1 and in equation (2).

$$EROI_{pou} = \frac{\text{energy output from extraction and processing}}{\text{direct and indirect energy inputs}} \quad (2)$$

### 3.2. National-level EROI

Up to this point we have described EROI types that analyse a single energy resource, modifying the system boundary to include more or less stages along the energy transformation chain, but still for a single energy source. When various energy resources are examined within certain geographical limits, another type of EROI emerges: a societal or national-level EROI. The most recent attempt to calculate a societal EROI ( $EROI_{soc}$ ) was undertaken by Lambert et al. (2013; 2014). These calculations are based on price and energy intensity information. However there is a danger that using a price based approach introduces distortions as prices also represent non-physical factors at play in the economy and hence does not necessarily reflect resource availability or accessibility<sup>1</sup>.

The calculation in Lambert et al. (2013) estimates the average EROI for all energy supply technologies deployed by a nation.  $EROI_{soc}$  is calculated by dividing the average energy obtained per dollar of spending (summed over different fuel inputs to the economy) by the primary energy needed to obtain one dollar's worth of economic production (see equation (3)). Lambert et al. (2013) found that low income countries have low  $EROI_{soc}$  of 2-9:1, whereas lower and upper middle income countries have relatively higher  $EROI_{soc}$  of 3-29:1, varying between energy exporting and energy importing countries.

$$EROI_{soc} = \frac{GDP}{E_T} * \sum_{n=1}^i \eta_i \frac{E_{U_i}}{E_{P_i}} \quad (3)$$

Where  $E_U$  is the energy content of a unit of energy [MJ],  $E_P$  is the price of that unit of energy [USD],  $E_T$  is total energy consumed by a society [MJ], GDP is Gross Domestic Product [USD] and  $\eta$  is the ratio of net energy contribution (Lambert et al., 2014).

There have been previous attempts to calculating the net energy for a country. Peet et al. (1987) calculated the net energy (i.e. energy output less energy inputs) obtained by the New Zealand economy between 1947 and 1983. Input-Output (IO) data was used to obtain energy intensities. This combined with money expenditures allowed the calculation of embodied energy flows for the country's electricity and oil supplies. An array of numbers (one for each energy source) was generated however rather than a single EROI for the country.

Additionally Leach (1975) in his critique paper effectively calculates a net energy ratio for the UK for 1968. This calculation was used to illustrate potential problems in the calculation and interpretation of energy return figures. However, as was acknowledged and further pointed out (Slessor, 1976), it is how EROI is changing over time that is of interest as can occur in a time of rapid transition from one energy technology to another. Methodological considerations are discussed in the next section.

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<sup>1</sup> If there is an assumption of perfectly competitive markets, prices can also be assumed to reflect quality, accessibility and scarcity. However, the underlying assumptions for perfectly competitive markets can be contested.

## 4. Methodological Issues

Within the EROI literature there are some persistent methodological and transparency issues, which are considered here. These are: how to define boundaries of analysis, how to account for embodied energy, how to deal with temporality and how to account for energy quality. These are largely the same issues that Leach (1975) identified as being discussed in a NEA workshop held in August 1975 at Stanford, California. An additional methodological issue specific of a national-level EROI will also be analysed and that is cross-border trade.

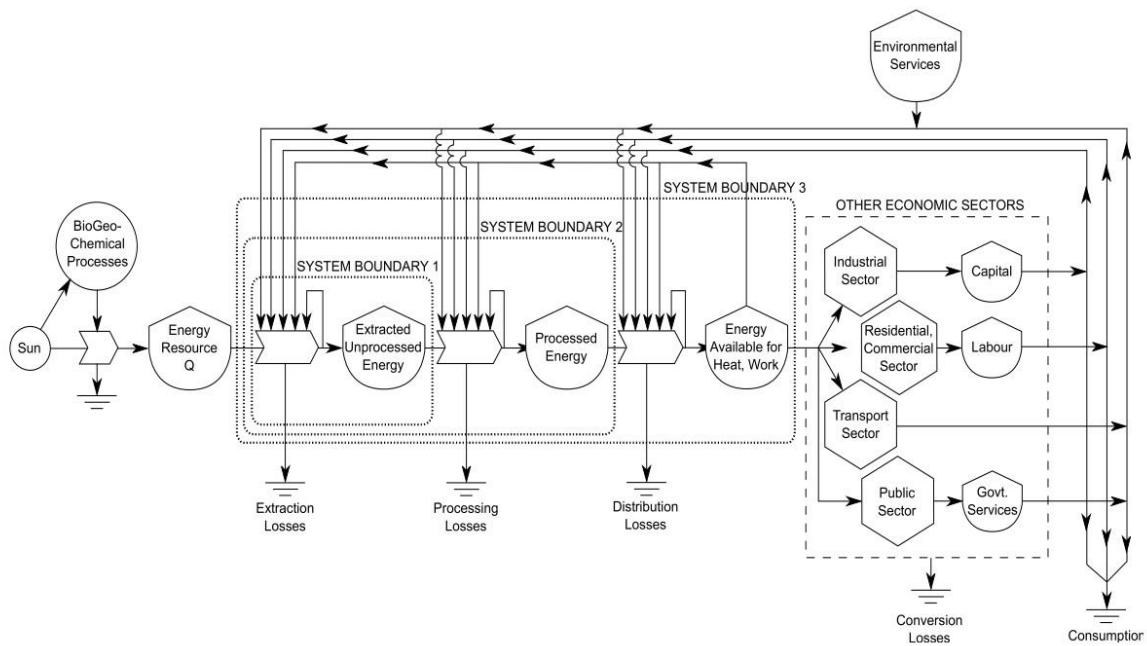
### 4.1. Boundary

There are three main considerations when assessing boundaries for EROI, which are summarised here and then discussed in more detail. Firstly how far along the energy processing chain to go: primary energy, energy carriers, final consumption or useful energy (useful work)? Second, a decision is required as to the inclusions for energy inputs at each of the energy chain steps under analysis, i.e. embodied energy in capital equipment, operation and maintenance energy, energy consumed by the labour force, etc. Third, a consideration is required as to the range of energy sources that will be analysed and within what geographical limits, e.g. a single energy source in a particular extraction/capture site or multiple energy sources in a given region.

Maybe one of the few areas of consensus regarding the system boundary definition is around the accounting start point for EROI. EROI “assumes that the energy in the ground (or coming from the sun) is not to be counted as an input” (Herendeen, 2004, p. 284). Therefore, EROI accounts for energy inputs once they have been either extracted or harnessed for human purposes. This is depicted in Figure 2 along with the system boundary classification framework developed by Murphy et al. (2011). Note that this start point of accounting for energy contrasts with the approach of another assessment tool: Life Cycle Analysis (LCA). In LCA the energy that is present in the environment or the energy source is the start point for accounting in measures of, for instance, cumulative energy demand.

Moving to the question of the extent of energy inputs included at each stage of the energy chain, the first step is to include the direct energy and material (as embodied energy) inputs. Continuing with the example of an oil well, the direct energy and material inputs would be the energy required to operate the oil rig and the embodied energy in the oil rig itself. However, this boundary can be expanded to include the indirect energy and material inputs, i.e. the inputs required to make the initial inputs. For example the energy required to make the oil rig itself and the embodied energy in the materials used to make the oil rig.

Even more expansions to the boundary that determines the energy inputs can be made. For example, indirect labour consumption can be included, as well as the consumption of auxiliary services and the environmental impacts of the production of direct and indirect energy and materials. Furthermore, considering that the accounting of these energy inputs is usually limited to energy that passes through the market, certain free flows of energy are not typically accounted for (e.g. solar heating, photosynthesis, etc.).



**Figure 2. EROI system boundaries**

Source: Figure taken from Murphy et al. (2011, p. 1891).

Finally, there is the issue of how many energy sources are being analysed and within which geographical limits. Many EROI studies focus on a single energy source in a single location, for example the EROI of a specific oil well or coal mine. Some other studies focus on a single energy source but within a bigger geographical boundary, for example a whole country or region. Murphy et al. (2011) and Hall et al. (2014) have undertaken detailed reviews of published EROI values for various energy sources and regions. There are very few temporal EROI studies. For example Brandt (2011) conducted an EROI investigation of oil in California over the period 1955 to 2005.

A transparent and clear boundary definition is critical in being able to make comparisons between EROI studies. This is something that has been lacking somewhat in EROI publications (Murphy et al., 2011). Special consideration is required as to what energy inputs are included. This aspect can be particularly challenging in conducting EROI calculations.

#### *4.2. Accounting for energy inputs: top-down or bottom-up*

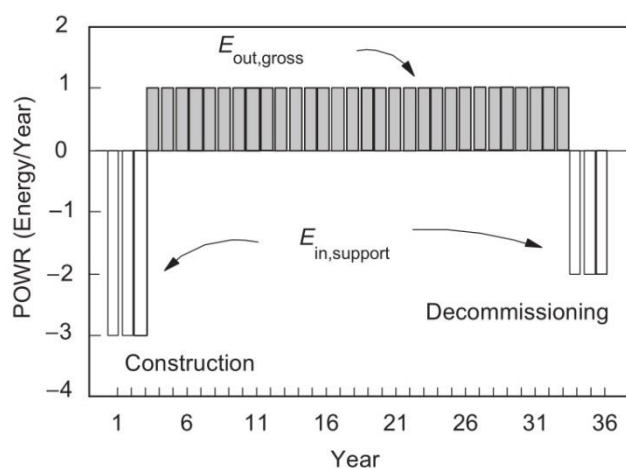
Depending on the chosen boundaries, certain methodology can be preferred for the accounting of energy inputs. The two main methodologies used are Input-Output (IO) and process analysis. The former is a top-down approach, whilst the latter is a bottom-up approach. They both have positive and negative aspects which should be recognised when analysing the results derived from one or the other (Murphy et al., 2011).

IO is more appropriate when the boundary is expanded to multiple processes e.g. when considering activities at a national level. This is due to greater capacity to display interrelationships across economic sectors (Murphy et al., 2011). However the necessary level of aggregation means it is difficult to conduct analysis on a particular energy source, or technology or specific location.

Process analysis is more appropriate when assessing a single energy source through a few clearly defined processing stages (Murphy et al., 2011). However, following Arvesen and Hertwich (2015), if using LCA in process analysis, care is needed to ensure that boundary conditions are consistent with the EROI calculation.

### 4.3. Temporality

Most EROI studies compute  $EROI_{std}$  to focus on a single energy source on a particular location, using a process based method. The timing of energy inputs and energy outputs over the useful life of the supply technology is important, particularly since there typically high energy inputs at the beginning (construction) and at the end (decommissioning) of the life of the energy extraction or capture location. This is represented in Figure 3 and discussed in detail for the case of phot-voltaic panels by Dale (2012).



**Figure 3. Temporality of energy inputs and outputs**

Source: Taken from Herendeen (2004, p. 285).

However, when the boundary is expanded over larger geographical spaces, a pragmatic approach is required as collecting such information for each and every energy supply technology would be immensely time consuming and expensive. One pragmatic solution is to assume that energy inputs at different points in time will be balanced out, since not all energy extraction or capture projects will be at the same stage of development. However, as Murphy et al. (2011, p. 1893) point out “this assumption would be accurate only if the system is in ‘steady state’, i.e., not growing or shrinking.”

### 4.4. Accounting for energy quality

How to account for the differences in energy quality of the different energy sources has been a persistent methodological issue in energy analysis and hence also for conducting net energy analysis (NEA). There are, in general, two approaches for accounting for differences in energy quality: price-based and physical units (Murphy et al., 2011). The price-based approach is used more often, even though it rests on the unrealistic assumptions of competitive markets and lack of accounting for externalities. The physical units approach on



the other hand, is used less often. We argue that this might be a result of its complexity, lack of readily available data and objectives of the analyses.

#### 4.5. Cross-border trade

When expanding the geographical boundary to a specific country, imports should be added and exports should be subtracted from the  $EROI_{nat}$  calculation. The latter is a straightforward procedure in the sense that exported energy is removed from the energy outputs (numerator), since that energy is not used within that country. However, accounting for energy imports can be more challenging, considering that the specific inputs required to produce that energy are geographically located elsewhere and fall within a completely different technological and resource context.

The following section will describe two approaches for calculating national-level EROI, and will specify how we have decided to deal with the methodological issues discussed in this section. Our choices do not intend, however, to point towards final solutions to these methodological issues, but rather contribute to the discussion from a country wide perspective.

### 5. National-level EROI – methods and data

#### 5.1. Overview

Two different approaches for calculating national-level EROI ( $EROI_{nat}$ ) will be described in this section. But first, we will discuss how they deal with the aforementioned methodological issues. In both approaches, the system boundary is defined as the first stage of extraction/capture of energy sources. In terms of most energy reporting, this means Total Primary Energy Supply (TPES). The choosing of this system boundary is related to the purposes of this exploratory paper, but it could be expanded as part of future research.

The boundary for determining the energy inputs is defined as direct and indirect energy and embodied energy inputs, following the convention for standard EROI studies. Regarding number of energy sources and geographical limits, these correspond to a national territory and all the energy sources used within that territory. Therefore,  $EROI_{nat}$  will reflect the aggregated and weighted values of EROI for all energy sources used within that territory. This means that imports and exports (trade) of energy will be accounted for.

Furthermore, in relation to energy quality we are not making any specific adjustments, and aggregation between energy sources is possible because of the traditional conversion factors used in most energy reporting. Finally, in relation to temporality, we assume that given it is a national-level indicator and therefore different energy sources are aggregated, the patterns of energy inputs will be averaged out, and therefore accounting for  $EROI_{nat}$  in any given year broadly reflects the whole country's EROI across all energy sources irrespective of the stage of development of specific energy extraction and capture projects<sup>2</sup>.

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<sup>2</sup> This might be a particularly troublesome assumption in a context of climate change, where the need to invest in particular energy sources in a short period of time is a real possibility. In these sort of periods, EROI values would be very low.

## 5.2. First approach: System dynamics (GEMBA) based calculation

### 5.2.1. Methodology

The first approach is based on physical flows. It can theoretically be calculated using process analysis to obtain the EROI of each energy source used in a country (imported sources would need to be calculated in addition), and then adding them up proportionally to the particular fuel mix of that country. It would have the advantage of being very detailed and having the possibility to assess specific technological innovations, but it would involve a disproportionate amount of work to undertake in practice.

Therefore, we have opted to use global values. These can be obtained from the EROI time-series values generated through the GEMBA model (Dale et al., 2012a, 2012b) for each energy source to the fuel mix of a given country for each year in the time-series, as in equation (4).

$$\sum GE_i * N_i \quad (4)$$

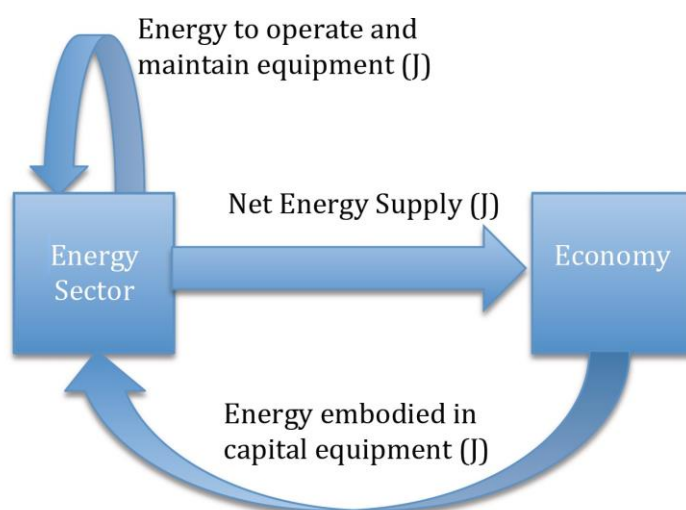
Where:

$i$  = energy source

$GE_i$  = EROI value from GEMBA model for energy source  $i$

$N_i$  = percentage of energy source  $i$  in the energy mix

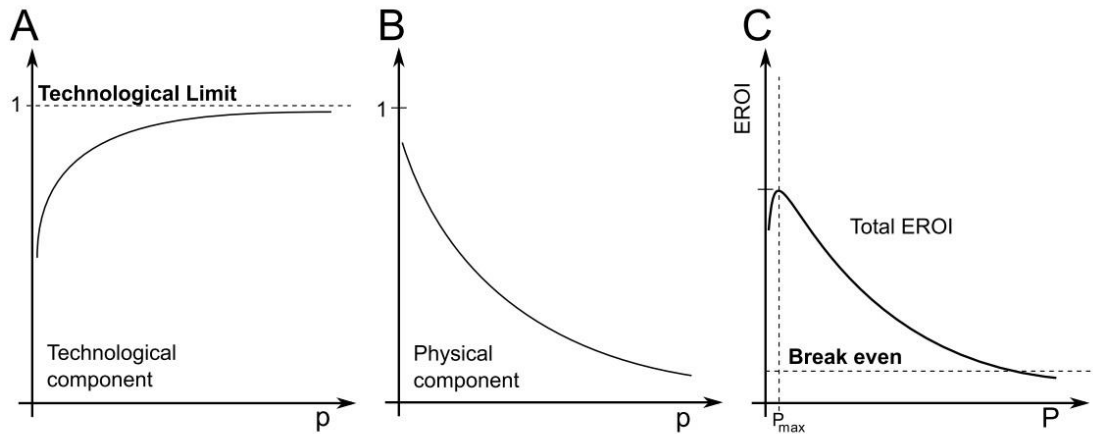
Figure 4 gives an overview of the energy-economy system on which GEMBA is based. Part of the energy output from the energy sector needs to be used to run and maintain exploitation equipment resulting in the lower “net” energy supply to the wider economy. The economy uses some of that net energy supply to manufacture and supply replacement/additional capital equipment and machinery in order to generate the required energy output. All flows are quantified in joules.



**Figure 4. Overview of an energy-economy system**

The EROI values generated from the GEMBA model are derived from a dynamic function that depends on a technological component and a physical component (see Figure 5). The

former represents technological advancements that occur with increased cumulative production (i.e. technological learning, experience), but within strict thermodynamic laws (Dale et al., 2011). The latter represents declining physical resource quality, assuming that the resources that offer the best returns (based on factors such as energy density, ease of accessibility and proximity to demand centres) are exploited first (Dale et al., 2011).



**Figure 5. Dynamic EROI function**

Notes:  $p$  (in Plot A and B) represents cumulative production normalized to the size of the URR (ultimately recoverable resource).  $P$  (in Plot C) represents total cumulative production.

This approach implies a very significant assumption: that the global average EROI values for the different energy sources apply similarly for any country, i.e. country specific energy temporal, spatial and technological development characteristics are not accounted for. Therefore, the country-level derived GEMBA- $EROI_{nat}$  comparisons will depend only on their energy mix, which is a big shortcoming in this alternative. In other words, the EROI values for different energy sources will be the same despite their country of origin; hence there are no differentiated values for traded energy.

Furthermore, GEMBA is an energy-economy simulation model based on the system dynamics modelling technique. The purpose of system dynamic models is to explore the behaviour of a system rather than to predict particular values; hence it is important to keep in mind that  $EROI_{nat}$  calculated using this approach can only be used for indicative temporal analysis rather than determining exact values. Additionally, the GEMBA model is set up to look at primary production specifically, which complies with the system boundary defined here.

Therefore, following on from the points above, the EROI values for part of the GEMBA modelled period are to be viewed as a very approximate baseline for comparison. A key advantage in using this approach is that it removes the potential distortions that can be present when taking a monetary based approach. In other words, it puts the calculation of  $EROI_{nat}$  on a wholly physical basis. In this sense, a comparison with  $EROI_{soc}$  as defined by Lambert et al. (2013) could provide some insights as to differences between monetary and purely physical approaches.

### 5.2.2. Data requirements

The data will be the EROI time-series values for each fuel from the GEMBA model and energy mix for particular countries in publications such as the annual report “World Energy Outlook” published by the International Energy Agency (IEA, 2012). The EROI time-series data have been provided by Michael Carbajales-Dale’s model (Dale et al., 2012a, 2012b) for the period 1800-2200 and for 12 different energy sources. For oil and gas there is a distinction between conventional and unconventional (tar sands, shale) sources in the GEMBA model.

### 5.3. Second approach: Input-Output based calculation

#### 5.3.1. Methodology

Like many other energy analysis techniques, energy IO analysis was developed in the 1970s driven by the oil price shock of the time (Casler & Wilbur, 1984). It has been mainly used to quantify energy flows through the different economic sectors (see for example Bullard & Herendeen, 1975; Bullard, Penner, & Pilati, 1978; Wright, 1974). However, to the best of our knowledge, it has not been used to directly calculate a national-level EROI value. Perhaps the study by Peet et al. (1987) is the closest one, but it focused on the oil and electricity sectors only.

Using matrix algebra, an energy extension on IO data can be used to calculate  $EROI_{nat}$ . In general, this approach aims to track all energy requirements of the energy sector, both direct and indirect. It does so by using a whole economy’s transaction matrix to allocate energy sales and purchases to every industry, and then track down the paths that lead to the energy industry itself. Therefore, it can be considered a mixed approach between purely physical and purely monetary approaches.

Furthermore, MRIO (Multi-Regional Input-Output) data is used because it can take into account the specific contexts of international primary energy trade. MRIO data and environmental extensions to it have been used before to calculate CO<sub>2</sub> emissions associated with consumption (Barrett et al., 2013). It has proven to be a very good tool for tracing CO<sub>2</sub> embodied in goods and services consumed in a specific country, even if the CO<sub>2</sub> was emitted elsewhere. This approach attempts to do something similar, but tracing the energy and embodied energy flows used by a country’s energy sector.

As stated above, the system boundary is defined at the extraction stage; therefore equation (5)<sup>3</sup> is consistent with equation (1).

$$EROI_{nat} = \frac{E_{ex}}{E_{in}} \quad (5)$$

Where:

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<sup>3</sup> From this point we will be describing this IO approach in relation to the UK (i.e. calculating  $EROI_{nat,UK}$ ).

$E_{ex}$  = net energy outputs from extraction/capture (or energy output from extraction in equation (1))

$E_{in}$  = direct and indirect energy inputs (as in equation (1))

In a context of national territories and traditional energy statistics  $E_{ex}$  can be further defined as:

$$E_{ex} = E_T - E_{dE} \quad (6)$$

Where:

$E_T$  = total primary energy supply (TPES)

$E_{dE}$  = total energy sector's own use from the UK and the RoW (rest of the world) used to extract/capture UK TPES

In this sense,  $E_{ex}$  can be easily calculated without using MRIO data (e.g. using IEA data). However, in order to give methodological consistency to this approach, we have decided to calculate it using MRIO data (see Appendix A for a detailed explanation and Appendix B for a numerical example). It should be noted that the energy sector's own use only accounts for reported numbers to energy agencies and energy flows that go through the market, but it could leave out some important unreported energy uses.

Similarly, in a context of national territories and MRIO data,  $E_{in}$  can be further defined as:

$$E_{in} = E_{dE} + E_{iE} \quad (7)$$

Where:

$E_{iE}$  = total indirect energy from UK and RoW used to extract/capture UK TPES

Therefore, equation (5) can be rewritten as:

$$EROI_{nat} = \frac{E_T - E_{dE}}{E_{dE} + E_{iE}} \quad (8)$$

A detailed explanation of the matrix algebra IO procedure used to calculate the indirect energy requirements can be found in Appendix A and a numerical example in Appendix B.

The construction of the MRIO dataset that we are suggesting to use is based on the direct impact coefficient approach (or energy intensity approach). It has the advantage of providing specific information on traded energy and having readily available data to work with. However, it lacks detail about specific technologies because they are indirectly represented in the transaction matrix through prices.

Another alternative would be to use a hybrid-unit approach in the construction of MRIO datasets. It would have the advantage of being able to use physical units only. Guevara (2014) constructed a hybrid-unit matrix for Portugal using IEA (International Energy Agency) data, with the aim of tracing different energy flows and performing a decomposition

analysis. However, to the best of our knowledge, MRIO data is not available under this hybrid-unit approach.

### 5.3.2. Data requirements

IO data on energy use, which can be obtained from EXIOBASE (Wood et al., 2014) which covers 43 countries and 5 RoW regions to cover the whole world. It also contains environmental extensions that include 16 “Nature Inputs”<sup>4</sup> which are the energy inputs relevant for this paper because they correspond to TPES. In relation to industries, it contains 5 energy industries/sectors<sup>5</sup> relevant for our boundary definition.

Although the environmental extension is in physical values (MJ), the main IO table is based on monetary values. This can be considered a drawback of this dataset, which uses a direct impact coefficient approach when compared to a hybrid-unit approach. However, its use is justified by data availability (there are no MRIO energy extended databases that we know of that use a hybrid-unit approach) and unit consistency.

## 6. Results and discussion

Draft results to be presented at the European Society for Ecological Economics (ESEE) 2015 conference: Transformations (June 30 – July 3, Leeds, UK).

## 7. Conclusions

This paper examines methodologically two alternative approaches to calculating  $EROI_{nat}$  for national economies. On one hand, a novel application of the EROI data results obtained from a system dynamics model (GEMBA model from Dale (2012b)). On the other hand, an approach based on Input-Output analysis. The system dynamics approach can be an interesting indicative and comparative tool, particularly for long time periods; however, it is limited in its accuracy given the nature of system dynamics models. The IO approach is, to the best of our knowledge, a novel application of MRIO datasets which has been enabled by the advances in IO data gathering. In this sense, it is an initial step that we hope can be built upon.

The relevance of these alternatives lies on their potential to inform national-level energy policy making on resource depletion and technological change in the nation’s energy sector, but also in the nations where they import energy from. Moreover, the dynamic of this relationship could be studied if  $EROI_{nat}$  time series are built. Additionally, EROI and exergy and useful work analysis can be considered complementary approaches, and when analysed

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<sup>4</sup> Anthracite; Coking Coal; Other Bituminous Coal; Sub-Bituminous Coal; Lignite/Brown Coal; Peat; Crude petroleum and services related to crude oil extraction, excluding surveying; Natural gas and services related to natural gas extraction, excluding surveying; Natural Gas Liquids; Other Hydrocarbons; Uranium and thorium ores; Electricity by hydro; Electricity by wind; Electricity by solar photovoltaic; Electricity by solar thermal; Electricity by tide, wave, ocean; Electricity by Geothermal; Energy inputs to cultivated biomass; Other sources.

<sup>5</sup> Mining of coal and lignite, extraction of peat; Extraction of crude petroleum and services related to crude oil extraction, excluding surveying; Extraction of natural gas and services related to natural gas extraction, excluding surveying; Extraction, liquefaction, and regasification of other petroleum and gaseous materials; Mining of uranium and thorium ores.

together can reveal future possibilities or limits to efficiency improvements in every step of the energy chain.

Finally, for future research and expansion of the methodology proposed here, particularly the IO approach, it seems reasonable that the energy returned to society (numerator) could be measured by the useful work output of that economy, as calculated through exergy efficiency analyses. However, it is less clear how the energy invested (denominator) for a national economy should be measured in exergy and useful work terms. Other strands of future research that become available through the IO approach include analysing the effect of particular energy sources and of subsequent conversion stages on a country's EROI.

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## APPENDIX A

*A note on notation:* A bold lower case letter represents a vector. A bold capital letter represents a matrix. Non-bold lower case and capital letter represent scalars. A vector with a “hat” ( $\hat{\phantom{x}}$ ) represents a diagonal matrix, whose diagonal elements are the elements of the vector. An apostrophe on a vector or a matrix represents the vector or matrix transpose.  $\mathbf{I}$  is called the identity matrix, and is a matrix of zeros whose diagonal is made of ones.

Consider the two dimensional transaction’s matrix  $\mathbf{Z}$  (Figure 6), which displays sales by each industry in rows and the columns represent purchases by each industry. In other words, reading across a row reveals which other industries a single industry sells to and reading down a column reveals who a single industry buys from in order to make its product output. A single element,  $z_{ij}$ , within  $\mathbf{Z}$  represents the contributions from the  $i^{\text{th}}$  supplying sector to the  $j^{\text{th}}$  producing sector in an economy. The  $\mathbf{Z}$  matrix is in monetary units.

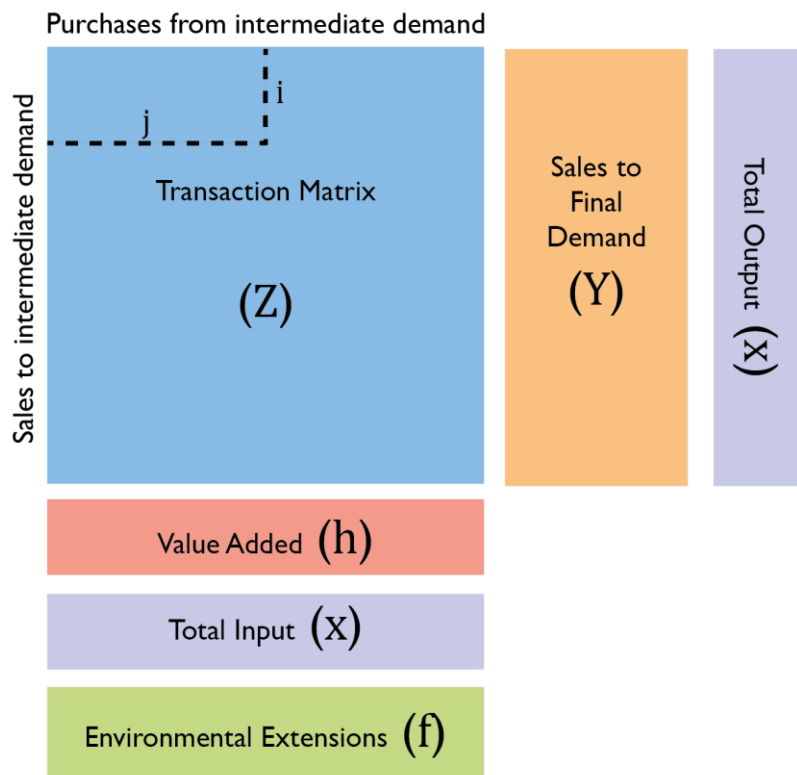


Figure 6. Basic structure of a Leontief Input-Output system

Reading across the table, the total output ( $x_i$ ) of a particular sector can be expressed as:

$$x_i = z_{i1} + z_{i2} + \dots + z_{in} + y_i \quad (9)$$

where  $y_i$  is the final demand for that product produced by the particular sector. Essentially, the IO framework shows that the total output of a sector can be shown to be the result of its intermediate and final demand. Similarly if a column of the IO table is considered, the total input of a sector is shown to be the result of its intermediate demand and the value added in profits and wages ( $h$ ). The sum across total output ( $x$ ) and total input ( $x$ ) will be equal.



If each element,  $z_{ij}$ , along row  $i$  is divided by the output  $x_j$ , associated with the corresponding column  $j$  it is found in, then each element in  $\mathbf{Z}$  can be replaced with:

$$\mathbf{a}_{ij} = \frac{\mathbf{z}_{ij}}{\mathbf{x}_j} \quad (10)$$

forming a new matrix  $\mathbf{A}$ , known as the direct requirements matrix. Element  $\mathbf{a}_{ij}$  is therefore the input as a proportion of all the inputs in the production recipe of that product.

Equation (10) can be re-written as:

$$\mathbf{z}_{ij} = \mathbf{a}_{ij}\mathbf{x}_j \quad (11)$$

Substituting for (11) in (9) forms:

$$\mathbf{x}_i = \mathbf{a}_{i1}\mathbf{x}_1 + \mathbf{a}_{i2}\mathbf{x}_2 + \dots + \mathbf{a}_{in}\mathbf{x}_n + \mathbf{y}_i \quad (12)$$

Which, if written in matrix notation is  $\mathbf{x} = \mathbf{Ax} + \mathbf{y}$ . Solving for  $\mathbf{y}$  gives:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y} \quad (13)$$

(13) is known as the Leontief equation and describes output  $\mathbf{x}$  as a function of final demand  $\mathbf{y}$ .  $\mathbf{I}$  is the identity matrix, and  $\mathbf{A}$  is the technical coefficient matrix showing the inter-industry requirements.  $(\mathbf{I} - \mathbf{A})^{-1}$  is known as the Leontief inverse (denoted hereafter as  $\mathbf{L}$ ). Therefore (13) can be re-written as:

$$\mathbf{x} = \mathbf{Ly} \quad (14)$$

Consider a row vector  $\mathbf{f}$  of annual energy required by each industrial sector (an environmental extension in Figure 6). Then it is possible to calculate the energy intensity ( $\mathbf{e}$ ) by dividing the total energy input of each sector by total sector output ( $\mathbf{x}$ ), in terms of joules per pound for example, as follows:

$$\mathbf{e} = \mathbf{fx}^{-1} \quad (15)$$

In other words,  $\mathbf{e}$  is the coefficient vector representing energy per unit of output.

Multiplying both sides of (14) by  $\mathbf{e}$  gives:

$$\mathbf{ex} = \mathbf{eLy} \quad (16)$$

and from (15) we simplify (16) to:

$$\mathbf{f} = \mathbf{eLy} \quad (17)$$

However, we need the result ( $\mathbf{f}$ ) as a flow matrix ( $\mathbf{F}$ ) and so we use the diagonalised  $\hat{\mathbf{e}}$  and  $\hat{\mathbf{y}}$ :

$$\mathbf{F} = \hat{\mathbf{e}}\mathbf{L}\hat{\mathbf{y}} \quad (18)$$

$\mathbf{F}$  is primary energy in matrix form, allowing the use of energy from the full supply chain of extraction/capture to be determined.  $\mathbf{F}$  is calculated by pre-multiplying  $\mathbf{L}$  by energy per unit of output and post-multiplying by final demand. Energy is reallocated from extraction/capture sectors to the sectors that use this primary energy.

The next step is to identify the energy flows for a specific country, here the UK:

$\mathbf{Z}$  contains multi regional data (it is a Multi-Region Input Output Table “MRIO” Table). Let  $m$  be the set of sectors drawn from numbers 1 to  $m$  and including sectors  $c$  to  $e$ , which are the energy sectors.

$$\{m \mid m \in \mathbb{N}_1, m = 1, 2, \dots, c, \dots, e, \dots, m\} \quad (19)$$

Let  $n$  be the set of countries drawn from numbers 1 to  $n$  and including country  $k$  representing the UK.

$$\{n \mid n \in \mathbb{N}_1, n = 1, 2, \dots, k, \dots, n\} \quad (20)$$

The dimensions of  $\mathbf{Z}$  are  $m \times n$ , where  $i$  represents the set of row numbers from 1 to  $mn$  and  $j$  represents the set of column numbers from 1 to  $mn$ .

$$\{i \mid i \in \mathbb{N}_1, i = 1, 2, \dots, mn\} \quad (21)$$

$$\{j \mid j \in \mathbb{N}_1, j = 1, 2, \dots, mn\} \quad (22)$$

$E_T$  is the sum of the UK’s TPES and can be found from the row vector  $\mathbf{f}$  and is the sum of the elements in position  $1 + (k - 1)m$  to  $km$ . This is the energy used in UK sectors only.

$$E_T = \sum_{j=1+(k-1)m}^{j=km} \mathbf{f} \quad (23)$$

$E_{UKdE}$  is the direct energy from both the rest of the world and the UK that is used to extract/capture UK energy. For this calculation we need to track the sum of all supply chains that trace energy from any of the energy sectors directly to the UK energy industry. For example, the energy from Norwegian oil used to extract UK coal is included, but the energy from Norwegian oil used to make machinery that is used by the UK coal extracting sector is excluded because this is not a direct path.

To calculate the sum of the direct paths we first find the direct UK energy used to satisfy total final demand for energy products ( $\mathbf{ey}$ ), then we calculate the sum of the energy paths that end up in UK energy products that are one step or further along the supply chain. To calculate this second part, we find the difference between the total UK energy footprint, and the total energy footprint if there were zero paths from any energy sectors to the UK. This difference is the sum of all energy paths to a UK product that are one step or further along the supply chain. This is represented by a new  $\mathbf{Z}^0$  matrix where zeros replace the

expenditure of UK energy sectors on all other energy inputs. We do this one country at a time to avoid counting any indirect paths.

Let  $j^0$  be a subset of  $j$  such that it only contains those column elements that represent the UK energy sectors. Let  $i^0$  be a subset of  $i$  such that it contains those row elements that represent the energy sectors from the UK and all other countries.

$$\{j^0 \subset j \mid j^0 = c + (k-1)m, \dots, e + (k-1)m\} \quad (24)$$

$$\{i^0 \subset i \mid i^0 = c + (n-1)m, \dots, e + (n-1)m\} \quad (25)$$

Element  $z_{ij}^0$  from the matrix  $\mathbf{Z}^0$  is zero when both  $i$  belongs to the set of  $i^0$  and  $j$  belongs to the set of  $j^0$ . Otherwise,  $z_{ij}^0$  is the same as  $z_{ij}$ .

$$\begin{cases} z_{ij}^0 = z_{ij} \leftrightarrow i \notin i^0 \cdot j \notin j^0 \\ z_{ij}^0 = 0 \leftrightarrow i \in i^0 \cdot j \in j^0 \end{cases} \quad (26)$$

We next calculate the corresponding  $\mathbf{A}^0$ ,  $\mathbf{L}^0$  and  $\mathbf{F}^0$  matrices following equations (10), (14) and (18). Now

$$E_{UKdE} = \mathbf{e}\mathbf{y} + \sum_{i=1}^{i=mn} \sum_{j=1}^{j=mn} \mathbf{F} - \mathbf{F}^0 \quad (27)$$

$E_{in}$  is the sum of the direct and indirect Energy used to make UK energy. This can be calculated as the sum of the columns representing UK energy sectors.

$$E_{in} = \sum_{i=1}^{i=mn} \sum_{j=c+(k-1)m}^{j=e+(k-1)m} \mathbf{F} \quad (28)$$

## APPENDIX B

A numerical example of the above is provided here. Please note that the numbers used here are purely for illustration and do not reflect real data.

Let's assume that Table 1 is a MRIO Table, and the different components of the table are colour coded and comparable with Figure 6. The units of the MRIO Table are in Table 2. In this example, following from equation (19),  $m$  is 4 and  $c$  and  $e$  correspond to the third column (only 1 energy sector). Also in this example, following from equation (20),  $n$  is 2 (two countries), where  $k$  is 1 (the UK).

**Table 1. Example of a MRIO Table**

Z		UK	UK	UK	UK	RoW	RoW	RoW	RoW	UK	RoW	
		Agri	Manu	Energy	Service	Agri	Manu	Energy	Service	y	y	x
UK	Agri	100	30	10	2	3	5	7	4	500	5	666
UK	Manu	20	200	20	30	5	4	5	3	300	2	589
UK	Energy	40	50	500	200	6	3	4	2	300	5	1110
UK	Service	10	20	20	300	6	2	3	2	400	3	766
RoW	Agri	30	20	10	1	1000	20	20	20	30	600	1751
RoW	Manu	5	50	2	3	100	2500	40	30	30	400	3160
RoW	Energy	10	10	30	20	200	400	4000	1000	20	800	6490
RoW	Service	1	5	1	40	50	30	30	2000	10	1000	3167
	h	450	204	517	170	381	196	2381	106			
	x	666	589	1110	766	1751	3160	6490	3167			
Energy	f	300	200	500	100	400	500	1000	200			

**Table 2. Units of the MRIO Table**

Array	Units
Z	£
y	£
x	£
h	£
f	J

The **A** matrix is presented in Table 3. Each element of the **A** matrix is calculated using equation (10).

**Table 3. A matrix**

A	0.15	0.05	0.01	0.00	0.00	0.00	0.00	0.00
	0.03	0.34	0.02	0.04	0.00	0.00	0.00	0.00
	0.06	0.08	0.45	0.26	0.00	0.00	0.00	0.00
	0.02	0.03	0.02	0.39	0.00	0.00	0.00	0.00
	0.05	0.03	0.01	0.00	0.57	0.01	0.00	0.01
	0.01	0.08	0.00	0.00	0.06	0.79	0.01	0.01
	0.02	0.02	0.03	0.03	0.11	0.13	0.62	0.32
	0.00	0.01	0.00	0.05	0.03	0.01	0.00	0.63

Table 4 shows the  $(\mathbf{I} - \mathbf{A})$  matrix. It is calculated by subtracting each element of the identity matrix (**I**) with each element of the **A** matrix. Its inverse, the **L** matrix, is presented in Table 5.

**Table 4. (I-A) matrix**

I-A	0.85	- 0.05	- 0.01	- 0.00	- 0.00	- 0.00	- 0.00	- 0.00
	- 0.03	0.66	- 0.02	- 0.04	- 0.00	- 0.00	- 0.00	- 0.00
	- 0.06	- 0.08	0.55	- 0.26	- 0.00	- 0.00	- 0.00	- 0.00
	- 0.02	- 0.03	- 0.02	0.61	- 0.00	- 0.00	- 0.00	- 0.00
	- 0.05	- 0.03	- 0.01	- 0.00	0.43	- 0.01	- 0.00	- 0.01
	- 0.01	- 0.08	- 0.00	- 0.00	- 0.06	0.21	- 0.01	- 0.01
	- 0.02	- 0.02	- 0.03	- 0.03	- 0.11	- 0.13	0.38	- 0.32
	- 0.00	- 0.01	- 0.00	- 0.05	- 0.03	- 0.01	- 0.00	0.37

**Table 5. The Leontief inverse (L matrix)**

inv(I-A)	1.18	0.10	0.02	0.02	0.01	0.01	0.00	0.01
	0.06	1.54	0.06	0.12	0.02	0.01	0.00	0.01
	0.16	0.30	1.86	0.82	0.03	0.02	0.01	0.01
	0.04	0.10	0.06	1.68	0.02	0.01	0.00	0.01
	0.14	0.15	0.05	0.04	2.36	0.09	0.02	0.06
	0.11	0.69	0.06	0.13	0.69	4.88	0.09	0.22
	0.16	0.44	0.19	0.44	1.11	1.77	2.67	2.36
	0.03	0.09	0.02	0.25	0.22	0.16	0.04	2.75

$e$  is calculated following equation (15), and  $\hat{e}$  and  $\hat{y}$  are presented in Table 6 and Table 7.

**Table 6. Diagonalised e**

$e^{\wedge}$	0.45	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.34	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.45	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.13	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.23	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.16	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06

**Table 7. Diagonalised y**

$y^{\wedge}$	505.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	302.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	305.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	403.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	630.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	430.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	820.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1010.00

We obtain  $F$  as in equation (18), which is shown in Table 8.

**Table 8. F matrix**

$e^{\wedge}Ly^{\wedge}$	269.08	13.30	3.26	4.09	2.57	2.48	1.46	3.76
	10.53	157.43	5.79	16.98	3.28	1.99	1.10	2.90
	36.00	40.48	255.50	148.75	8.18	3.64	2.10	5.35
	2.53	3.90	2.36	88.20	1.40	0.49	0.30	0.82
	15.66	10.56	3.40	3.99	339.04	8.82	4.11	14.42
	9.01	33.01	2.93	8.05	69.18	331.89	11.48	34.46
	12.69	20.63	8.98	27.47	108.08	117.14	337.90	367.11
	0.88	1.63	0.43	6.48	8.64	4.25	1.97	175.72

Finally, we calculate  $Z^0$  following equations (24), (25) and (26), which is shown in Table 9 (the cells highlighted in yellow show the zeroed paths), together with the corresponding  $A^0$ ,  $L^0$  and  $F^0$  matrices following equations (10), (14) and (18). These are shown in Table 10, Table 11 and Table 12 respectively. As indicated in Appendix A, we do the “zeroing” one country at a time, but in this example, for simplicity reasons, we are showing it as one step.

**Table 9. Z zero matrix**

$Z^0$		UK	UK	UK	UK	RoW	RoW	RoW	RoW	UK	RoW	
		Agri	Manu	Energy	Service	Agri	Manu	Energy	Service	y	y	x
UK	Agri	100	30	10	2	3	5	7	4	500	5	666
UK	Manu	20	200	20	30	5	4	5	3	300	2	589
UK	Energy	40	50	0	200	6	3	4	2	300	5	610
UK	Service	10	20	20	300	6	2	3	2	400	3	766
RoW	Agri	30	20	10	1	1000	20	20	20	30	600	1751
RoW	Manu	5	50	2	3	100	2500	40	30	30	400	3160
RoW	Energy	10	10	0	20	200	400	4000	1000	20	800	6460
RoW	Service	1	5	1	40	50	30	30	2000	10	1000	3167
	h	450	204	1047	170	381	196	2381	106			
	x	666	589	1110	766	1751	3160	6490	3167			
Energy	f	300	200	500	100	400	500	1000	200			

**Table 10. A zero matrix**

$A^0$	0.15	0.05	0.01	0.00	0.00	0.00	0.00	0.00
	0.03	0.34	0.02	0.04	0.00	0.00	0.00	0.00
	0.06	0.08	-	0.26	0.00	0.00	0.00	0.00
	0.02	0.03	0.02	0.39	0.00	0.00	0.00	0.00
	0.05	0.03	0.01	0.00	0.57	0.01	0.00	0.01
	0.01	0.08	0.00	0.00	0.06	0.79	0.01	0.01
	0.02	0.02	-	0.03	0.11	0.13	0.62	0.32
	0.00	0.01	0.00	0.05	0.03	0.01	0.00	0.63

**Table 11. L zero matrix**

$\text{inv}(I-A)^0$	1.18	0.10	0.01	0.02	0.01	0.01	0.00	0.01
	0.06	1.53	0.03	0.11	0.01	0.01	0.00	0.01
	0.09	0.16	1.01	0.45	0.02	0.01	0.00	0.01
	0.04	0.09	0.03	1.66	0.02	0.01	0.00	0.01
	0.13	0.15	0.03	0.03	2.36	0.09	0.02	0.06
	0.11	0.69	0.03	0.11	0.69	4.88	0.09	0.22
	0.15	0.42	0.03	0.37	1.11	1.77	2.67	2.36
	0.03	0.08	0.01	0.25	0.22	0.16	0.04	2.75

**Table 12. F zero matrix**

$e^{\Lambda} y^0$	268.87	13.06	1.76	3.22	2.52	2.45	1.45	3.73
	10.16	157.01	3.14	15.44	3.20	1.95	1.07	2.84
	19.59	22.03	139.02	80.94	4.45	1.98	1.14	2.91
	2.38	3.73	1.28	87.57	1.36	0.47	0.29	0.80
	15.44	10.30	1.81	3.06	338.99	8.80	4.10	14.39
	8.80	32.78	1.48	7.20	69.13	331.87	11.47	34.42
	11.63	19.44	1.45	23.08	107.83	117.03	337.84	366.96
	0.85	1.60	0.21	6.36	8.64	4.25	1.97	175.71

The  $EROI_{nat,UK}$  that we obtain from this example, using equations (5), (6), (7), (23), (27) and (28) is 3.43.

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